IMPROVED ANALYTICAL ANALYSIS OF NOISE FIGURES IN HEMT MIXERS

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Abstract— **An analytical analysis of noise figures in HEMT mixers is presented. Improved method was developed to evaluate the noise correlation. A simplified analytical expression for noise figure of HEMT mixer was also derived. The contribution of each noise source at various frequencies was represented in a phasor form to permit physical understanding of noise mechanism. The modeled noise figure was compared with the measured data, yielding a good agreement.**

I. Introduction

PHEMT mixers are one of the most promising candidates for mm-wave MMIC mixers[1]. Potential for conversion gain and easy integration capability with other circuits such as LNA's and VCO's makes this type of mixers very suitable for low-cost mm-wave receivers. There have been a number of analysis methods for FET mixers[2,3]. They can be categorized into analytical and numerical analysis. Although numerical analysis can be general and more accurate, it does not offer physical insight into mixing mechanism. Analytical method, on the other hand, allows one to understand the role of each nonlinear element and noise contributions from each noise source inside the device. This kind of information is valuable in optimizing the mixer performance in conjunction with the device parameters.

The analytical expressions for conversion gain have been derived for HEMT mixers by the author in [2]. This work extends the analysis presented in [2] to derive analytical expressions for noise figure. There were a limited number of theoretical works on FET mixer noise[4-6]. Numerical approaches were used in [4][6]. The work by Tie and Aitchison[5] derived an analytical solution, but the method was based on time average values of each nonlinear element, resulting in oversimplified results. In this work, noise correlations between different mixing frequencies as well as different noise sources were considered. In addition, the noise correlation calculation presented in [4] was modified to get better results. Simplified analytical expression for noise figure of HEMT mixer was derived and the phasor diagram analysis was performed. The analysis method was verified with measured results.

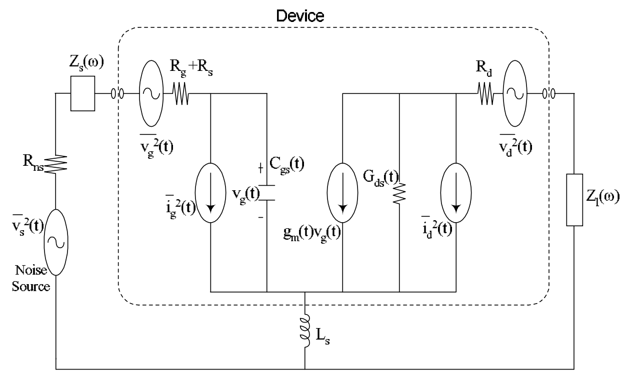

Fig. 1. Simplified equivalent circuit of HEMT mixers for the calculation of mixer noise figure

II. Analysis Method

The equivalent circuit of HEMT's together with associated noise sources is shown in Fig. 1. In order to simplify the analysis, the following assumptions were made.

1. The gate terminal is short-circuited at the IF frequency and the harmonics. Only the RF and IM(image) frequency components are present at the gate.
2. Order of nonlinearity is limited to 2 except for the output conductance (G_{ds}) whose nonlinearity is limited to one. Mixing with an order greater than 3 was thus neglected.

Noise power is present at RF and IM frequencies at the gate, and RF, IM and IF frequencies at the drain. The noise sources considered include the extrinsic thermal noise source due to access resistances (R_s, R_g, and R_d in Fig. 1), and intrinsic noise sources due to the device operation ($\overline{i_g^2}$ and $\overline{i_d^2}$ in Fig. 1). Noise coefficients(P, R, C) were used to represent two intrinsic noise sources and their correlation as in [4][7].

The noise figure can then be written in the form

$$F = 1 + \frac{\overline{i_{gg}^2} + \overline{i_i^2} + \overline{i_{dd}^2}}{\overline{i_s^2}} \tag{1}$$

where $\overline{i_{gg}^2}$ represents the thermal noise power at the load generated by R_s and R_g at IF frequency, and $\overline{i_{dd}^2}$ is the noise power from R_d. $\overline{i_i^2}$ is the noise component due to intrinsic noise sources and is therefore correlated. Finally, $\overline{i_s^2}$ is the noise power coming from the source through the input matching circuit. Each noise current component was calculated independently using a superposition principle. First, $\overline{i_s^2}$ was calculated using the conversion gain formula derived in [2]. The only difference from the conversion gain calculations is the presence of sources at two frequencies instead of one, one at RF and the other at IM. $\overline{i_s^2}$ can then be represented as a linear combination of RF and IM components as :

$$\overline{i_s^2} = \overline{|G_{RF}|^2} \cdot \overline{v_{sRF}^2} + \overline{|G_{IM}|^2} \cdot \overline{v_{sIM}^2} \qquad (2)$$

where

$$G_{RF} = \frac{1 + [j\omega_{IM} Z_s(\omega_{IM}) - \omega_{IM}^2 L_s] C_{gs_0} + j\omega_{IM} L_s g_{m_0}}{\eta} \cdot \psi \qquad (3)$$

$$G_{IM} = \frac{1 + [j\omega_{RF} Z_s(\omega_{RF}) - \omega_{RF}^2 L_s] C_{gs_0} + j\omega_{RF} L_s g_{m_0}}{\eta} \cdot \psi^* \qquad (4)$$

$$\psi = -g_{m_1}^* \frac{Z_l(\omega_{IF})}{1 + G_{ds_0} Z_l(\omega_{IF})} - \frac{Z_l(\omega_{IF})}{1 + G_{ds_0} Z_l(\omega_{IF})} \Bigg\{ - g_{m_0} \mathcal{F}_1^*(\alpha) - g_{m_1}^*[1 - \mathcal{F}_0(\alpha)] \\ g_{m_1}\mathcal{F}_2^*(\alpha) - \sum_{l=-N, |l|\geq 2}^{N} g_{m_l}(-1)^{l+1}\mathcal{F}_{l+1}^*(\alpha) \Bigg\}, \qquad (5)$$

and $\alpha = \{G_{ds_1} Z_l(\omega)\}/\{1 + G_{ds_0} Z_l(\omega)\}$.

$$\mathcal{F}_n(z) \equiv \sum_{k=0}^{\infty} \binom{2k+n}{k} z^{2k+n} \qquad (6)$$

G_{RF} is conversion gain from RF to IF frequency, and G_{IM} is that from IM to IF frequency.

The same approach can be applied to the calculation of $\overline{i_{gg}^2}$ with the sources replaced by $4KTB(R_s+R_g)$. $\overline{i_{dd}^2}$ is calculated using the linear transfer function from R_d to the load. The intrinsic noise component $\overline{i_i^2}$ was calculated by carefully taking correlation effects into account. The noise component due to i_g (in Fig. 1) was calculated using the conversion gain formula similar to $\overline{i_{gg}^2}$ and $\overline{i_s^2}$. Noise component due to i_d consists of the linear noise at IF frequencies and the nonlinear component at RF and IM frequencies downconverted to IF by G_{ds} nonlinearity. The total noise current at the load is the sum of the noise current components of i_g and i_d as

$$i_i = X i_{g_{RF}} + Y i_{g_{IM}} + x i_{d_{RF}} + y i_{d_{IF}} + z i_{d_{IM}} \qquad (7)$$

where $X = G_{RF} Z_s(\omega_{RF})$, $Y = G_{IM} Z_s(\omega_{IM})$. x, y and z are the transfer functions from i_d to load. The noise current power is then given by

$$\overline{i_i^2} = \{XX^*\overline{i_{g_{RF}}^2} + XY^*\overline{i_{g_{RF}} i_{g_{IM}}^*} + Xx^*\overline{i_{g_{RF}} i_{d_{RF}}^*} + Xy^*\overline{i_{g_{RF}} i_{d_{IF}}^*} + Xz^*\overline{i_{g_{RF}} i_{d_{IM}}^*}\} + \{YX^*\overline{i_{g_{IM}} i_{g_{RF}}^*} + \cdots\} + \cdots \qquad (8)$$

Eq. (6) contains correlation effects between i_g and i_d at various mixing frequencies. The noise correlation between m'th and n'th mixing frequencies of i_g, for example, was given in [4] as

$$\overline{i_{g_m} i_{g_n}^*} = 4KTB \frac{\omega_i C_{gs_i} \omega_j C_{gs_j}}{\sqrt{g_{m_i} g_{m_j}}} \sqrt{R_i R_j} \qquad (9)$$

The above expression consisted of harmonic components of g_m, C_{gs} and noise coefficients(P, R, C), making the evaluation easy. It is, however, not a true representation of the correlation. According to Dragone[8], the correlation between m'th and n'th mixing frequencies of i_g should be expressed as

$$\overline{i_{g_m} i_{g_n}^*} = 4KTB\omega_i\omega_j \left[\frac{C_{gs}^2(t)R(t)}{g_m(t)}\right]_{m-n} \qquad (10)$$

where $[\,]_{m-n}$ denotes $(m-n)'$th fourier coefficient.

The above equation can be either evaluated numerically by calculating the fourier components of $C_{gs}^2(t)/g_m(t) \cdot R(t)$ or expressed analytically in terms of harmonic components of g_m, C_{gs} after proper approximation. The latter is required for deriving analytical formula for noise figure.

III. Analysis, Simplification and Comparison

The analysis method was applied to a transconductance mixer shown in Fig. 2. An X-band MIC HEMT mixer was designed, fabricated and tested for this purpose. The noise parameters (F_{min}, R_n and Γ_{opt}) of the HEMT's were measured up to 40GHz at various V_g and V_d bias points, and the bias dependence of P, R and C were extracted as seen in Fig. 3. Using a table-based nonlinear model[1], single tone harmonic balance simulation was performed to evaluate the harmonic components of g_m, C_{gs} and G_{ds} at various LO power levels. Intrinsic noise components were evaluated following the analysis given in the previous section and the results

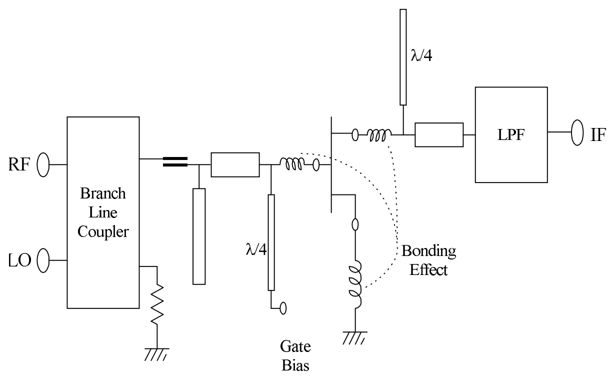

Fig. 2. Equivalent circuit schematic of gate mixer

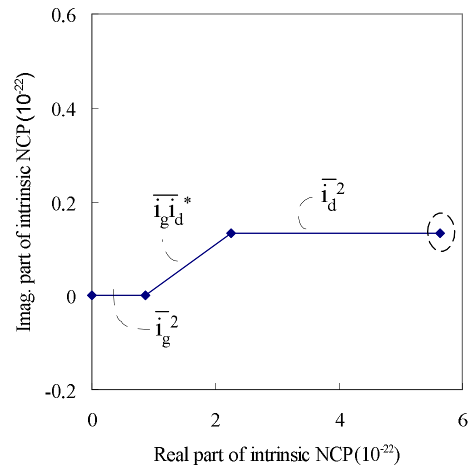

Fig. 3. Phasor diagram of total noise current power(NCP) due to $\overline{i_g^2}$, $\overline{i_d^2}$ and $\overline{i_g i_d^*}$ (P_{LO} = 3 dBm)

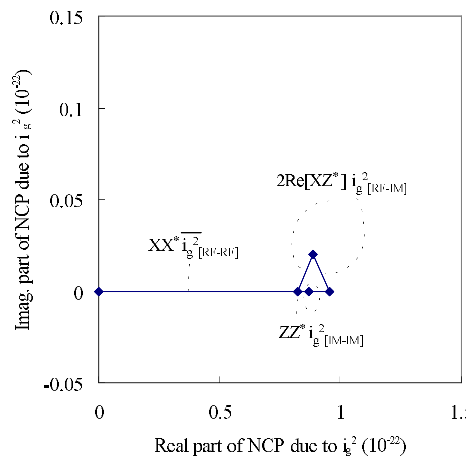

Fig. 4. Phasor diagram of $\overline{i_g^2}$ coming from each frequency components (P_{LO} = 3 dBm)

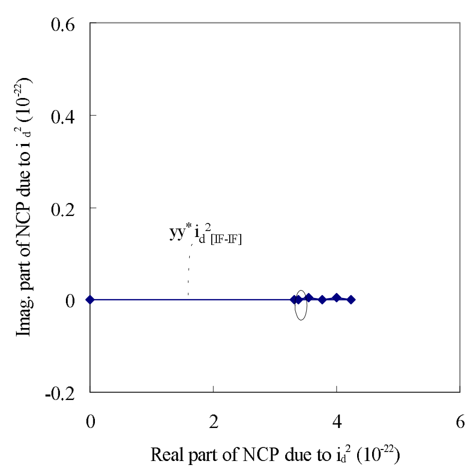

Fig. 5. Phasor diagram of $\overline{i_d^2}$ coming from each frequency components (P_{LO} = 3 dBm)

were represented in a phasor form in Fig. 3. Dominant components can be easily found for Fig. 4-6 The correlation effects increase the noise figure as seen in Fig. 3. After evaluating each component, a simplified analytical expression can be obtained in the form

$$F = 1 + \frac{(R_g + R_s)}{Re[Z_s(\omega)]} + \frac{XX^* \overline{i^2_{g[RF-RF]}} + yy^* \overline{i^2_{d[IF-IF]}} + 2Re[Xy^* \overline{i_g i_{d[RF-IF]}}]}{(\overline{|G_{RF}|^2} + \overline{|G_{IM}|^2}) \cdot \overline{v_s^2}} \quad (11)$$

The first term in the numerator of third term in (11) represents $\overline{i_g^2}$ noise from RF to IF. The second term is the linear noise of $\overline{i_d^2}$ and the third term represents the noise correlation between the RF component of i_g and the IF component of i_d.

In order to validate the analysis method, the modeled noise figure data was compared with measurements in Fig. 7. Good agreement was found between measured and modeled noise figure. The fit was good for a broad range of LO power levels. The noise figure calculated with the simplified formula (Eq. (11)) was within 0.6dB from the measurements. It can easily be employed as a first-pass design tool for HEMT mixers.

IV. Conclusions

An improved analytical analysis method for the evaluation of noise figures in HEMT mixers was developed. Noise correlation calculation presented in [4] was modified to allow better results. Simplified analytical expression for the noise figure was derived and contributions of each noise source and frequency were represented in a phasor form. This method permits an improved physical understanding of the nonlinear noise mechanism and

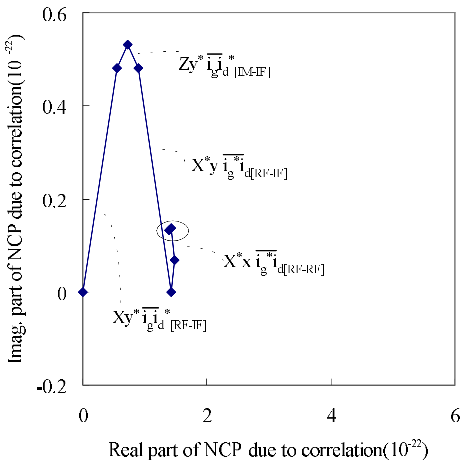

Fig. 6. Phasor diagram of $\overline{i_g i_d^*}$ (noise correlation) coming from each frequency components (P_{LO} = 3 dBm)

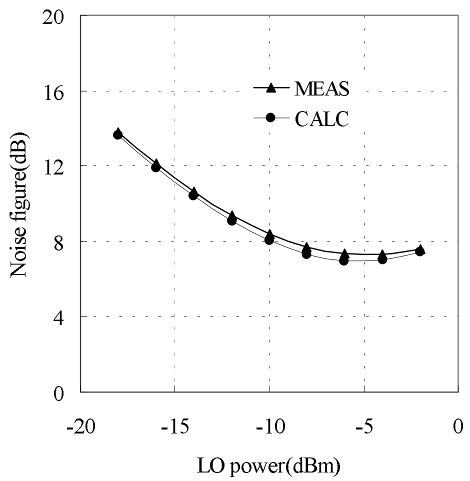

Fig. 7. Comparison of measured and modeled noise figures. Circle-dotted line represents the measured data, and solid line represents the calculated data using the new method. Improvement in the fit can be observed at high LO power levels.

provides a simple method for first-pass design of HEMT mixers. The model was validated with measurements.

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